Depth of Focus

by

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In 1963 at the Astronomical League's 17th Annual Convention, held in Orono, Maine, the late Robert E. Cox, who ran the "*Gleanings for ATMs*" column in Sky and Telescope, presented a paper on instrumental tolerances. This was subsequently published in the League's "*Proceedings*", and Sky & Telescope, "*Gleanings for ATMs*", Aug. 1964, pp 97-99.

Among the tolerances Cox discussed was depth of focus. Cox demonstrated that depth of focus was independent of aperture, and that telescopes of the same focal ratio possessed the same depth of focus, regardless of their focal length. The longer the f/ratio, the greater the depth of focus.

It was Cox's intention to show that while an ordinary rack and pinion focuser would be sufficiently accurate at f/15 or f/10, it would not be sufficiently precise at f/5. However the approach Cox followed was based on that of A.E. Conrady for the limits of *'focal adjustment'* on either side of the exact geometric focus. "*Applied Optics and Optical Design*", Dover 1957 p 136. The formula derived by Conrady is based on geometric optics, and whereas it is satisfactory for camera lenses is unsuitable for diffraction limited telescope optics. Conrady considered that there existed a zone.... *either side of the geometric focus that caused a difference to the optical path not exceeding the Rayleigh limit*. And to quote Cox - *That is within this range the difference between the longest and shortest optical paths leading to the image will not exceed one quarter of a wavelength of light. For visual work we can consider the wavelength as about 0.000022", the* λ *in Conrady's formula,*

$$\text{Focal Range} = \lambda / N' \sin^2 U'$$

N' *is the index of refraction of the medium in which the light is traveling, air in this case; therefore* $N' = 1$. U' *is the angle the edge ray makes with the optical axis. The sine of* U' *is* r/F, *and* $2 \sin U' = 2r/F$, *the reciprocal of which is the instrument's focal ratio,* $F/2r$. *Therefore, the formula simplifies to:*

$$\text{Focal Range} = 0.000088\ (\text{Focal Ratio})^2$$

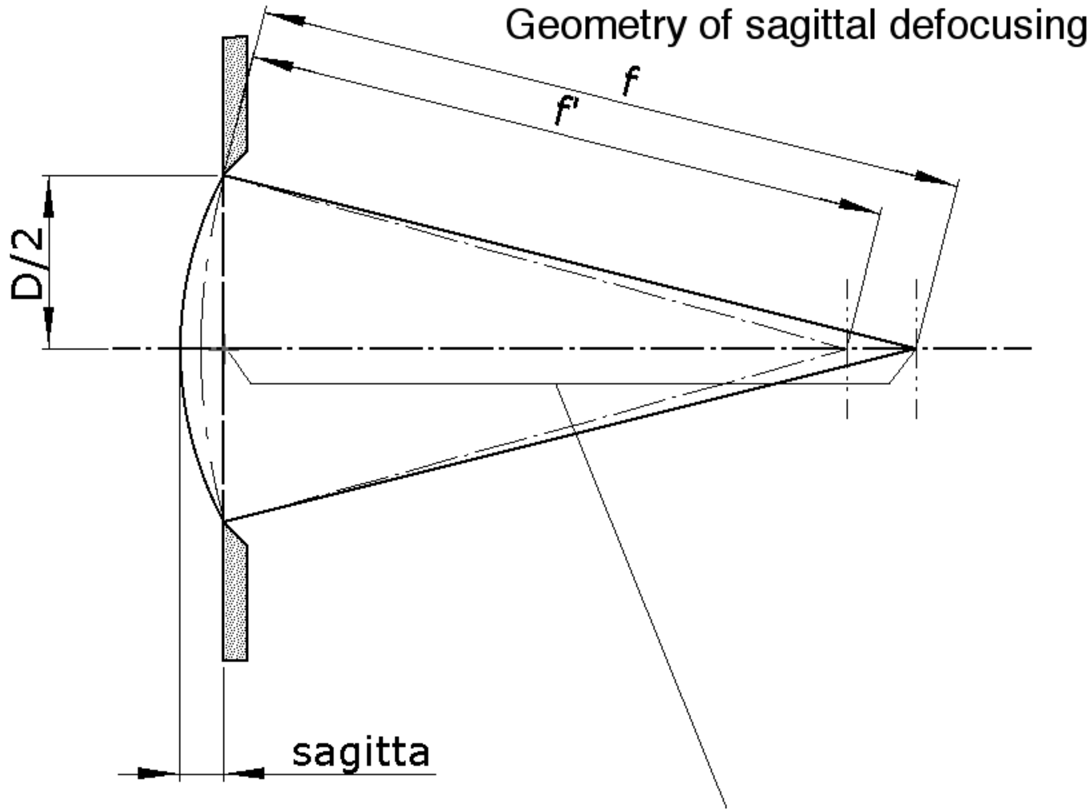
To understand the concept of depth of focus it is necessary to consider what is called defocusing aberration. For a given wavefront displacement at the aperture, (the defocusing aberration), there is a concommitent displacement of the focal plane. Twice this displacement (inside and outside the nominal focal plane or position of true focus) gives the depth of focus.

To derive the depth of focus the difference between the depths of a pair of wavefronts must be calculated. These are referred to as sagittal depths, the amount the wavefront is cupped, and if the focal distance is given as ‘f’, and the aperture as ‘D’, then, referring to the sagittal geometry diagram, by Pythagoras’ theorem:

sagitta:- $$s = f - \sqrt{f^2 - \left(\frac{D}{2}\right)^2}$$

The displaced wavefront may similarly be defined by:

sagitta prime: $$s' = f' - \sqrt{f'^2 - \left(\frac{D}{2}\right)^2}$$



$$\sqrt{f^2 - \left(\frac{D}{2}\right)^2}$$

The value $s - s'$ is known as the defocusing aberration, and may be defined in terms of the acceptable tolerance of the wavefront. If this is defined by "$\Delta n\lambda$" where λ is the wavelength then:

defocusing aberration: $s - s' \equiv \Delta n\lambda$

and the corresponding depth of focus:

$$(f - f') = \left\{ \left(\frac{s}{2} + \frac{D^2}{8s} \right) - \left(\frac{s'}{2} + \frac{D^2}{8s'} \right) \right\}$$ *see appendix derivation (i)

Chris Lord has run a calculation of this equation on an AppleWorksv6 spreadsheet, for f/ratios 4 to 30, and wavefront displacements, 1/10; 1/8; 1/4 & 1/2 wave of yellow light (22 micro-inches or 550nm). Depth of focus is given in inches. There is also a chart plotted over the range f/4 to f/24.

The depth of focus corresponding to a specific wavefront displacement is independent of the aperture, which for the purposes of the spreadsheet calculation was set at 1-inch. Conrady used 1/4 λ for the purposes of his calculations, but we hope you will realize that, in the case of very high quality optics, where the wavefront error is itself not more than 1/10 λ, the tolerable depth of focus is correspondingly reduced. Furthermore it should be born in mind that a wavefront displacement of 1/10 λ equates to a peak to valley (p-v) surface error of 1/20 λ, and so on.

A number of observers have remarked to Rodger Gordon, that when using their short f/ratio telescope to observer either the moon of planets, they found it necessary to refocus far more often than with long f/ratio telescopes under similar seeing conditions.

A displacement to the wavefront may also be introduced by seeing. Seeing can be modelled using Zernicke radial polynomials as descriptors of the wavefront effect. The first and second order expressions describe the tilt and the retardation of the wavefront respectively. (For a complete description of how Zernicke polynomials are used to describe wavefront aberrations see, "*Reflecting Telescope Optics*" Vol.1, Wilson R.N., Springer-Verlag 1996.)

If the tilt is contained within the depth of focus, image detail will remain in focus. If it is not so contained, image detail will be defocused, and there will be a perceptible loss of contrast and definition. Because the depth of focus is dependent on the wavefront errors of the optical system, and the focal ratio, it is therefore evident that more accurately figured optics, for a given f/ratio, are more susceptible to poor seeing, and for a given optical accuracy, short f/ratio telescopes are also more susceptible.

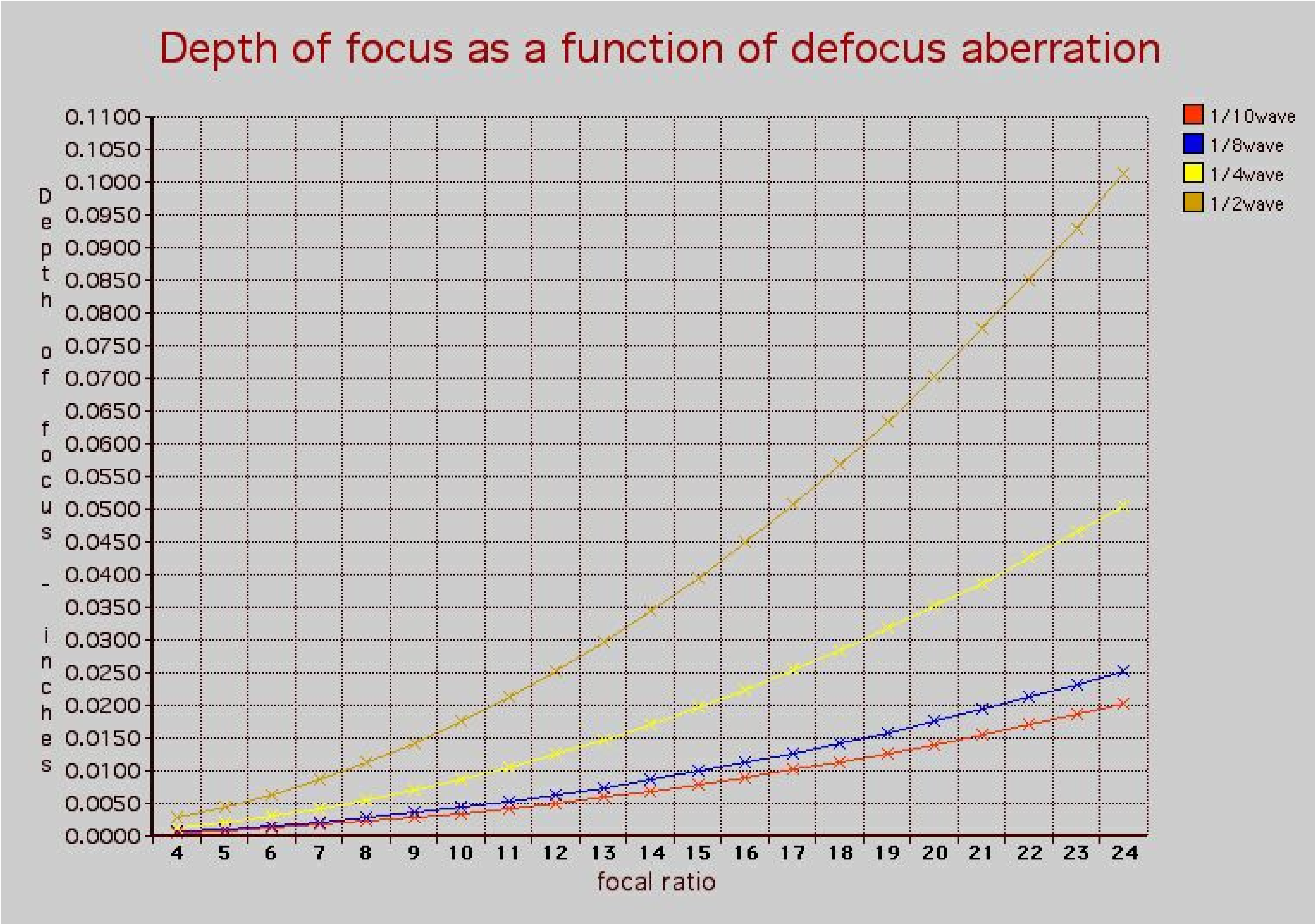
It can be seen from the table that the depth of focus for a 1/10 λ f/4 Newtonian is only $^{6}/_{10,000}$ inch, whereas depth of focus for a 1/10 λ f/10 Newtonian is $^{35}/_{10,000}$ inch, almost six times greater. Not only is focusing of an f/4 Newtonian far more critical, seeing continually takes the image in and out of focus because the depth of focus is so shallow.

We know from our own personal experience that having to continually refocus a large, fast f/ratio Newtonian is a trial. We also know that large (meaning 14-inches thru' 36-inches) aperture 'scopes are more seriously effected by poor seeing. This is because, from many experiments, air cell size is in large measure an indicator of how good or poor the seeing is.

When one is using a telescope with an aperture smaller than the air cells, the tendency will be for the definition to be good. When the air cells are smaller than the aperture of the telescope, and especially if those cells are moving fast, definition will be poorer. Air cells will have a slight difference of refractive index, and as they pass across the aperture, a displacement to the wavefront occurs. When no displacement is perceived seeing is said to be perfect. The probability of there being air cells larger than an aperture greater than 12-inches is vanishingly small. Couple this with the tendency of large Newtonians to have fast f/ratios, and you combine the worst of both scenarios.

F/#	1/10wave	1/8wave	1/4wave	1/2wave
4	0.0006	0.0007	0.0014	0.0028
5	0.0009	0.0011	0.0022	0.0044
6	0.0013	0.0016	0.0032	0.0063
7	0.0017	0.0021	0.0043	0.0086
8	0.0022	0.0028	0.0056	0.0112
9	0.0028	0.0036	0.0071	0.0142
10	0.0035	0.0044	0.0088	0.0176
11	0.0043	0.0053	0.0106	0.0213
12	0.0051	0.0063	0.0127	0.0253
13	0.0059	0.0074	0.0148	0.0297
14	0.0069	0.0086	0.0172	0.0344
15	0.0079	0.0099	0.0198	0.0395
16	0.0090	0.0112	0.0225	0.0450
17	0.0102	0.0127	0.0254	0.0508
18	0.0114	0.0142	0.0285	0.0570
19	0.0127	0.0159	0.0317	0.0635
20	0.0141	0.0176	0.0352	0.0703
21	0.0155	0.0194	0.0388	0.0775
22	0.0170	0.0213	0.0425	0.0851
23	0.0186	0.0232	0.0465	0.0930
24	0.0203	0.0253	0.0506	0.1013
25	0.0220	0.0275	0.0549	0.1099
26	0.0238	0.0297	0.0594	0.1188
27	0.0256	0.0320	0.0641	0.1281
28	0.0276	0.0345	0.0689	0.1378
29	0.0296	0.0370	0.0739	0.1478
30	0.0316	0.0395	0.0791	0.1582

The image of say an 18-inch f/4 Newtonian is far less likely to stay in focus, in Antoniadi seeing III, or even I-II, than a 10-inch f/10 Newtonian. Astronomers sometimes stop down the aperture of their telescope to combat poor or indifferent seeing, in an attempt to match the aperture to the size of the air cells, which range from a mere $^{1}/_{3}$-inch to a well over a foot, but are most commonly between 4 and 8-inches. Stopping down the aperture not only matches the aperture to the seeing, it increases the depth of focus thereby enabling a much more comfortable observing session because the observer no longer has to continually refocus the image. It has been our personal experience that seeing does tend to appear better in long f/ratio telescopes.


Depth of focus as a function of defocus aberration
Depth of focus - inches
0.1100
0.1050
0.1000
0.0950
0.0900
0.0850
0.0800
0.0750
0.0700
0.0650
0.0600
0.0550
0.0500
0.0450
0.0400
0.0350
0.0300
0.0250
0.0200
0.0150
0.0100
0.0050
0.0000
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
focal ratio
1/10wave
1/8wave
1/4wave
1/2wave

Amateur astronomers, particularly lunar and planetary observers have almost always preferred long f/ratio telescopes. There are a number of optical factors why this is so. For achromatic refractors the problem of secondary spectrum is lessened, and for apochromatics, spherochromatism is far less, and for Newtonians the minor axis of the secondary can be kept smaller. In each instance the effect is to minimize contrast loss. Couple this with greater depth of focus and the lower susceptibility to the effects of seeing, and it is no real surprise. And yet how many amateur astronomers must there be who, having been weened on a small aperture long f/ratio refractor or an f/8 Newtonian yearn for greater resolution, and develope aperture fever? Of course the assumption is that with the bigger aperture will come a corresponding increase in contrast and definition. But what they get in opting almost inevitably for the fast f/ratio Newtonian in the 18 to 24-inch or even 30-inch aperture range, is a decrease in image contrast, and a second rate hi-res 'scope. It can almost never be used at very high power (meaning over x40 per inch), the image has lower contrast due to the larger central obstruction, and it is badly effected by seeing, both in terms of the aperture effect and the miniscule depth of focus. In fact the only way to guarantee an improvement in contrast and definition is to fit an off axis stop, and to use it as an unobstructed reflector! Apart from the advantage, albeit marginal, over the $^{1}/_{5}$ to $^{1}/_{6}$ c - o of a mid-range Newtonian, in doing so, one is back to a medium effective aperture in the range 8 to 12 inches!

Another interesting facet emerges from this discussion. C17th and early C18th non-achromatic refractors had enormous focal ratios, in an attempt to reduce chromatic aberration. It was usual for the object glass to have an f/ratio anywhere between f/30 and f/100, and an aperture between 2 to 3-inches. For a 1/4 λ wavefront displacement an f/40 objective would have a depth of focus of 0.14 inches. In such a telescope the image would remain sensibly in focus through several turns of a rackmount's pinion. Rodger Gordon, having once owned an f/26.6 Schiefspiegler, recollects the wide focusing range through which objects remained in focus. His current 8-inch f/26 Cassegrain has a coarse rack and pinion focuser, and yet achieving a precise focus is easy because of the $^{60}/_{1,000}$-inch depth of focus. Using a Barlow lens increases the effective focal ratio and hence the depth of focus. It would evidently be worthwhile using a Barlow (of the highest quality obviously) and a longer focal length eyepiece to achieve high power, than a short focal length eyepiece. (Yet another reason for avoiding fast f/ratio apochromats and Newtonians).

The moral of all this, is that if you have a small(ish) first 'scope, say 2-inch f/15 to 5-inch f/12 achromatic refractor, or a 6-inch or 8-inch f/7 or f/8 Newtonian, and you want to upgrade your kit, do not think in terms of the big, fast Newtonian, especially a Dobsonian. If you want the ultimate telescope for lunar and planetary observing, think in terms of either a Maksutov-Newtonian, or a long focus mid range Newtonian, say either a 10-inch f/10 or a 12-inch f/8, or a Schiefspiegler.

Appendix:

*

the depth of focus is derived as follows:

$$\Delta f = f - f'$$

$$s = f - \sqrt{f^2 - \frac{D^2}{4}}$$

$$s' = f' - \sqrt{f'^2 - \frac{D^2}{4}}$$

$$\therefore s' - f' = -\sqrt{f'^2 - \frac{D^2}{4}}$$

$$\therefore (s' - f')^2 = f'^2 - \frac{D^2}{4}$$

$$\therefore s'^2 - 2f's' + f'^2 = f'^2 - \frac{D^2}{4}$$

$$\therefore s'^2 - 2f's' = -\frac{D^2}{4}$$

$$\therefore s' - 2f' = -\frac{D^2}{4s'}$$

$$\therefore f' = \frac{s'}{2} + \frac{D^2}{8s'}$$ & similarly: $$\therefore f = \frac{s}{2} + \frac{D^2}{8s'}$$

$\therefore$ depth of focus:

$$= (f - f') = \left\{ \left(\frac{s}{2} + \frac{D^2}{8s} \right) - \left(\frac{s'}{2} + \frac{D^2}{8s'} \right) \right\} \quad \text{(i)}$$

Appendix:

Note:

It can be shown that to a very high degree of precision depth of focus is proportional to the square of the f/ratio.

If the expression for sagitta is expanded using the Binomial theorem:

$$s \cong \frac{D^2}{8} - \frac{D^4}{128} + ...$$

neglecting higher order terms:

$$s - s' = \frac{D^2}{8}\left(\frac{f' - f}{f \ f'}\right)$$

and because

$$f \cong f',$$

then:

$$f . f' \cong f^2$$

$$\therefore s - s' \cong \frac{(f' - f)}{8.f/\#^2} \equiv \Delta n\lambda$$

hence:

$$f' - f \cong \Delta n\lambda.8.f/\#^2 \qquad \text{(ii)}$$

Appendix:

It can also be shown that depth of focus is not exactly independent of aperture, although the variance is negligible:

from: $$f' = \frac{s'}{2} + \frac{D^2}{8s'}$$

and: $$s' = s - \Delta n\lambda$$

$$\therefore \quad f' = \frac{(s - \Delta n\lambda)}{2} + \frac{D^2}{8(s - \Delta n\lambda)} \qquad \text{(iii)}$$

and from: $$s = f - \sqrt{f^2 - \frac{D^2}{4}}$$

$$\therefore \quad f' = \frac{\left(f - \sqrt{f^2 - \frac{D^2}{4}} - \Delta n\lambda \right)}{2} + \frac{D^2}{8\left(f - \sqrt{f^2 - \frac{D^2}{4}} - \Delta n\lambda \right)}$$

$$\therefore \quad f' = \frac{1}{2}\left\{ \left(f - \sqrt{f^2 - \frac{D^2}{4}} - \Delta n\lambda \right) + \frac{D^2}{4\left(f - \sqrt{f^2 - \frac{D^2}{4}} - \Delta n\lambda \right)} \right\}$$

and from: $$f = D.f/\#$$

Appendix:

$$\therefore \quad f' = \frac{1}{2}\left\{\left(D.f/\# - \sqrt{D^2.f/\#^2 - \frac{D^2}{4}} - \Delta n\lambda\right) + \frac{D^2}{4\left(D.f/\# - \sqrt{D^2.f/\#^2 - \frac{D^2}{4}} - \Delta n\lambda\right)}\right\} \qquad \text{(iv)}$$

$$\therefore \quad f - f' = D.f/\# - \frac{1}{2}\left\{\left(D.f/\# - \sqrt{D^2.f/\#^2 - \frac{D^2}{4}} - \Delta n\lambda\right) + \frac{D^2}{4\left(D.f/\# - \sqrt{D^2.f/\#^2 - \frac{D^2}{4}} - \Delta n\lambda\right)}\right\}$$

$$\therefore \quad f - f' = D\left\{f/\# - \frac{1}{2}\left(\left(f/\# - \sqrt{f/\#^2 - \frac{1}{4}} - \frac{\Delta n\lambda}{D}\right) + \frac{1}{4\left(f/\# - \sqrt{f/\#^2 - \frac{1}{4}} - \frac{\Delta n\lambda}{D}\right)}\right)\right\}$$

$$\therefore \quad f - f' = D\left\{f/\# - \frac{1}{2}\left(f/\#\left(1 - \sqrt{1 - \frac{1}{4f/\#^2}} - \frac{\Delta n\lambda}{D.f/\#}\right) + \frac{1}{4f/\#\left(1 - \sqrt{1 - \frac{1}{4f/\#^2}} - \frac{\Delta n\lambda}{D.f/\#}\right)}\right)\right\}$$

$$\therefore \quad f - f' = D\left\{ f/\# - \frac{1}{2} f/\# \left(\left(1 - \sqrt{1 - \frac{1}{4f/\#^2}} - \frac{\Delta n\lambda}{D.f/\#} \right) + \frac{1}{4f/\#^2 \left(1 - \sqrt{1 - \frac{1}{4f/\#^2}} - \frac{\Delta n\lambda}{D.f/\#} \right)} \right) \right\}$$

$$\therefore \quad f - f' = D.f/\# \left\{ 1 - \frac{1}{2} \left(\left(1 - \sqrt{1 - \frac{1}{4f/\#^2}} - \frac{\Delta n\lambda}{D.f/\#} \right) + \frac{1}{4f/\#^2 \left(1 - \sqrt{1 - \frac{1}{4f/\#^2}} - \frac{\Delta n\lambda}{D.f/\#} \right)} \right) \right\} \qquad \text{(v)}$$

Let:

$$k = 4f/\#^2 \left(1 - \sqrt{1 - \frac{1}{4f/\#^2}} - \frac{\Delta n\lambda}{D.f/\#} \right)$$

$$\therefore \quad f - f' = D.f/\# \left\{ 1 - \frac{1}{2} \left(\frac{k}{4f/\#^2} + \frac{1}{k} \right) \right\}$$

$$\therefore \quad f - f' = D.f/\# \left\{ 1 - \left(\frac{k}{8f/\#^2} + \frac{1}{2k} \right) \right\} \qquad \text{(vi)}$$

compare with the approximation:

$$f' - f \cong \Delta n\lambda.8.f/\#^2 \qquad \text{(ii)}$$

It is easy to demonstrate that depth of focus is not truly independent of aperture by calculating the differences between the true depth of focus based on the defocusing aberration as given by equation (vi) and the depth of focus derived from equation (ii) for a range of values of D.

Chris Lord has compiled three comparison tables showing the percentage error of the approximation for apertures 1-inch; 10-inch & 100-inch over a focal ratio range f/1 to f/30. Below f/4 the error is less than 1%, across this aperture range.

depth of focus %error_true minus approximation				
focal ratio	Aperture D = 1 - inch			
F/#	1/10wave	1/8wave	1/4wave	1/2wave
1	-23.758	-23.758	-23.755	-23.749
2	-4.942	-4.941	-4.937	-4.927
3	-2.127	-2.126	-2.119	-2.106
4	-1.180	-1.178	-1.170	-1.152
5	-0.747	-0.745	-0.734	-0.712
6	-0.513	-0.511	-0.497	-0.471
7	-0.372	-0.369	-0.353	-0.322
8	-0.280	-0.276	-0.259	-0.223
9	-0.216	-0.212	-0.192	-0.153
10	-0.170	-0.166	-0.144	-0.100
11	-0.136	-0.131	-0.107	-0.058
12	-0.109	-0.104	-0.078	-0.025
13	-0.088	-0.082	-0.054	0.003
14	-0.071	-0.065	-0.034	0.028
15	-0.057	-0.050	-0.017	0.049
16	-0.045	-0.038	-0.003	0.068
17	-0.035	-0.028	0.010	0.085
18	-0.026	-0.018	0.021	0.101
19	-0.019	-0.010	0.032	0.115
20	-0.012	-0.003	0.041	0.129
21	-0.006	0.004	0.050	0.142
22	-0.000	0.010	0.058	0.155
23	0.005	0.015	0.066	0.167
24	0.010	0.020	0.073	0.179
25	0.014	0.025	0.080	0.190
26	0.018	0.029	0.087	0.201
27	0.022	0.034	0.093	0.212
28	0.025	0.038	0.099	0.223
29	0.029	0.042	0.105	0.233
30	0.032	0.045	0.111	0.243

depth of focus %error_true minus approximation				
focal ratio	Aperture D = 10 - inches			
F/#	1/10wave	1/8wave	1/4wave	1/2wave
1	-23.760	-23.760	-23.760	-23.759
2	-4.945	-4.945	-4.945	-4.944
3	-2.132	-2.132	-2.131	-2.130
4	-1.187	-1.186	-1.186	-1.184
5	-0.755	-0.755	-0.754	-0.752
6	-0.523	-0.523	-0.521	-0.519
7	-0.383	-0.383	-0.381	-0.378
8	-0.293	-0.292	-0.290	-0.287
9	-0.230	-0.230	-0.228	-0.224
10	-0.186	-0.186	-0.183	-0.179
11	-0.153	-0.153	-0.150	-0.146
12	-0.128	-0.128	-0.125	-0.120
13	-0.109	-0.108	-0.105	-0.100
14	-0.093	-0.093	-0.090	-0.083
15	-0.081	-0.080	-0.077	-0.070
16	-0.070	-0.070	-0.066	-0.059
17	-0.062	-0.061	-0.057	-0.050
18	-0.055	-0.054	-0.050	-0.042
19	-0.049	-0.048	-0.044	-0.035
20	-0.043	-0.042	-0.038	-0.029
21	-0.039	-0.038	-0.033	-0.024
22	-0.035	-0.034	-0.029	-0.019
23	-0.031	-0.030	-0.025	-0.015
24	-0.028	-0.027	-0.022	-0.011
25	-0.026	-0.025	-0.019	-0.008
26	-0.023	-0.022	-0.016	-0.005
27	-0.021	-0.020	-0.014	-0.002
28	-0.019	-0.018	-0.012	0.001
29	-0.017	-0.016	-0.010	0.003
30	-0.016	-0.014	-0.008	0.006

depth of focus %error_true minus approximation				
focal ratio	Aperture D = 100 - inches			
F/#	1/10wave	1/8wave	1/4wave	1/2wave
1	-23.760	-23.760	-23.760	-23.760
2	-4.946	-4.946	-4.946	-4.946
3	-2.133	-2.133	-2.133	-2.132
4	-1.187	-1.187	-1.187	-1.187
5	-0.756	-0.756	-0.756	-0.756
6	-0.524	-0.524	-0.524	-0.523
7	-0.384	-0.384	-0.384	-0.384
8	-0.294	-0.294	-0.294	-0.293
9	-0.232	-0.232	-0.232	-0.231
10	-0.188	-0.188	-0.187	-0.187
11	-0.155	-0.155	-0.155	-0.154
12	-0.130	-0.130	-0.130	-0.129
13	-0.111	-0.111	-0.111	-0.110
14	-0.096	-0.095	-0.095	-0.095
15	-0.083	-0.083	-0.083	-0.082
16	-0.073	-0.073	-0.073	-0.072
17	-0.065	-0.065	-0.064	-0.063
18	-0.058	-0.058	-0.057	-0.056
19	-0.052	-0.052	-0.051	-0.050
20	-0.047	-0.046	-0.046	-0.045
21	-0.042	-0.042	-0.042	-0.041
22	-0.038	-0.038	-0.038	-0.037
23	-0.035	-0.035	-0.034	-0.033
24	-0.032	-0.032	-0.032	-0.030
25	-0.030	-0.029	-0.029	-0.028
26	-0.027	-0.027	-0.027	-0.025
27	-0.025	-0.025	-0.025	-0.023
28	-0.023	-0.023	-0.023	-0.021
29	-0.022	-0.022	-0.021	-0.020
30	-0.020	-0.020	-0.020	-0.018