CONTRAST & DEFINITION

C.J.R. Lord FRAS

#10, 2120, 4th. Street, Sanata Monica, California 90405

Introduction

Lord Rayleigh's investigations into the performance of optical systems led to the classic resolution formula based upon the complete separation of two Airy discs.[1] This was refined shortly afterwards by the observational empirical law formulated by the Rev. William Rutter Dawes of Ormskirk.[2] Both firmly established that the resolving power of a diffraction limited telescope was solely dependent on its aperture.

At the time the factors governing the resolution of detail on extended objects were imperfectly understood. At the turn of the C19th. Percival Lowell and W.H. Pickering [3,4] did much to sort out the confusing medley of interelated criteria, but it was not until the mid 1950's that the effects of both object and image contrast were evaluated.[5]

The effects of image contrast of adjacent features govern the resolution and in this paper I have endeavoured to establish a more comprehensive relatioship between aperture, image contrast, and definition; and to show that the Rayleigh limit is a special case analogous to the resolution of detail upon extended objects at the limit of contrast differentiation of the eye.

The defining ability of any optical imaging system is not merely a property of that system's aperture, it is governed by the Modulation Transfer Function (M.T.F.), the ratio of the object contrast to the contrast of that object in the image plane. The higher the MTF the greater the resolving power of that system. The MTF is not only limited by the aberrations within the optical system but also the earth's atmosphere. Haze, turbulence and differential refraction will each contribute to a reduction in the total performance of the telescope.

Definition of lunar and planetary details.

Lunar and planetary observers have always realized that the Rayleigh limit does not adequately define the limit to which they may discern both low and high contrast details upon the surfaces of these objects. For example, although a high contrast object like the Cassini Division in Saturn's rings is less than 1" arc in width at mean opposition, it was discovered using only a 62mm objective and may be barely resolved in a 44mm glass.[6] Such an instrument according to Dawes is only capable of resolving 3".0 or thereabouts. Even low contrast features such as Jovian spots, or Aerian oases, are often resolved well below the theoretical limits as established by Lord Rayleigh.

The reason for this is that neither the Rayleigh, nor Dawes' limits deal with these types of optical conditions. The Rayleigh limit defines the minimum separation for the complete resolution of a pair of equally bright Airy discs, that in practice must be approximately the 6th. magnitude and yellowish or white in hue.

Double star observers however frequently violate both the Rayleigh and Dawes' limits by taking advantage of the phenomena of "partial resolution" for an egg or figure-of-eight shaped star is obviously double or multiple, even though imperfectly separated.

This however has nothing to do with differences in contrast. The Dawes' and Rayleigh limits deal with diffraction phenomena of fairly bright point sources. An extended image may be regarded as a mosaic of diffraction discs although this is not a perfectly valid analogy. The ring system is so much fainter than the central spot of the Airy disc that they will not be at their full effective sizes.

It would be easier to explain the actual effect of definition if it were realised that it is not the extended object that is the special case but the double star. Imagine if you will a negative image of a barely resolved double star ideally meeting Lord Rayleigh's concept. In effect one would be observing detail upon an extended object (two black dots and rings against a white background) and the fainter these dots were against that background the less well they would be resolved. In fact when the contrast fell below approximately 5% most people would be incapable of resolving them.[14*]

By this simple but effective analogy I think you can see now that it is in fact only by virtue of its being of a higher contrast than the adjacent background that a particular detail may be seen at all! This may seem a statement of the obvious, but to apply Dawes' limit to planetary and lunar features is to overlook this obvious fact.

Resolving power - contrast function

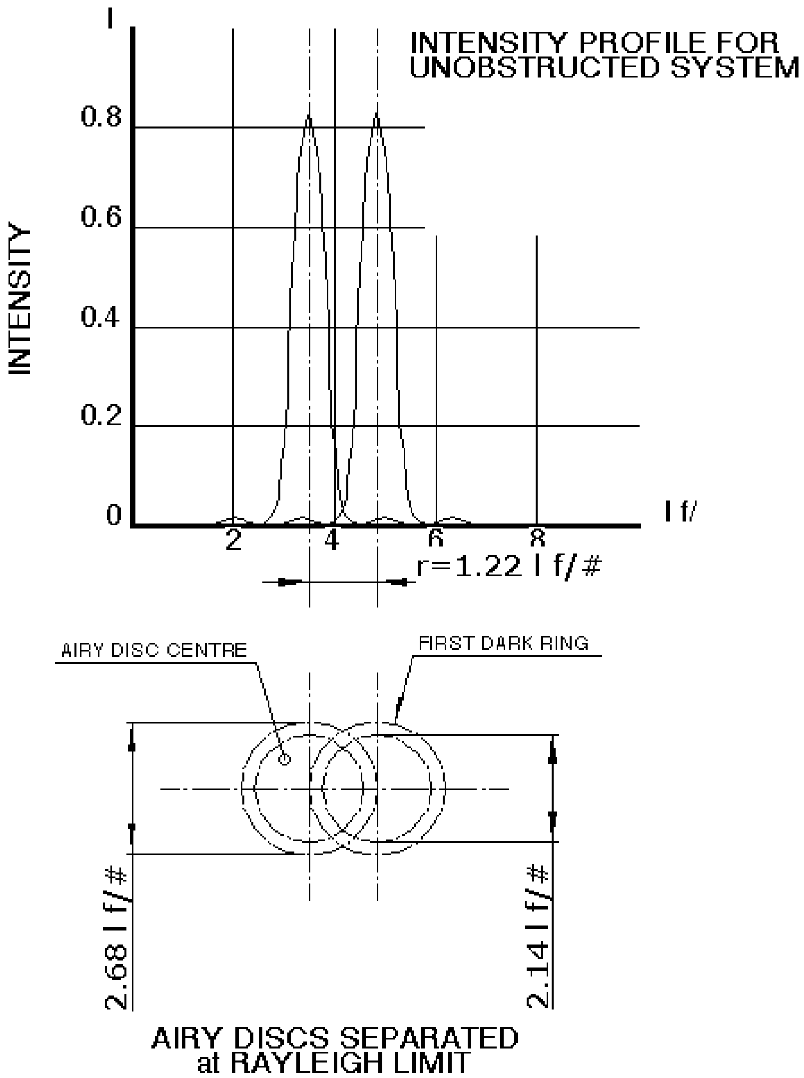
The relationship between resolution and object contrast is effected by the imaging ability of the telescope, the earth's atmosphere and visual acuity.[15] As Lowell pointed out in a paper communicated to the Royal Astronomical Society in November 15, 1905 [7] observers with particularly sensitive eyes, "susceptible to the action of light" were at a particular disadvantage when it came to the perception of form, compared to those observers possessed of acute vision. The distinction would seem to be a valid one, the two qualities being opposed, although its contribution to the sum total of the detail perceived a minor one; provided the observer in question has a trained eye.

It is frequently so that those possessed of sensitive vision find it difficult to perceive the subtle colours of planetary hues at first but eventually come to do so through practice. Whether this is because the eye becomes less sensitive, or that the experience afforded by many hours of observation contribute's to one's perception is unclear. The effect nevertheless exists.

It is possible to modify Lord Rayleigh's expression to cover all possible differences in image contrast.

The complete resolution of the Airy discs is given by a linear separation equal to the radius of the first ring.* (Refer to diagram)

** An experienced planetary observer may distinguish large contiguous areas some two or three times less contrasted than quoted. Furthermore between individuals contrast sensitivity at differing brightness levels varies considerably.*



FOR LIGHT WAVELENGTH λ = 550nm (A5500 YELLOW GREEN)

$\rho = 1.22 \,.\, 550.10^{-6}$ f/#

$r = 1.22.550.10^{-6}.206265/D_{mm}$

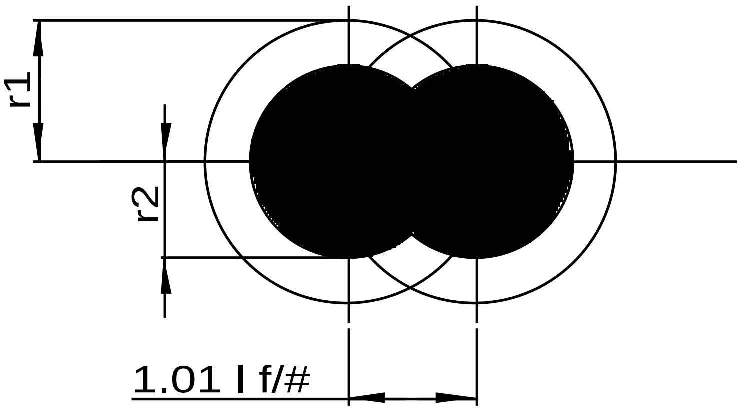
$r = 138/D_{mm}$ arc secs

** The size of the Airy disc centre is purely arbitary, for whilst theory can predict precisely where the zone of zero intensity lies in each interspace, it cannot define the point at which the intensity falls below the threshold of visibility. However a close approximation is $^1/_2\, r_1$*

This condition is valid only for extended object detail possessing a contrast difference of approximately 5%. Dawes' limit is an empirical relationship which readily translates to:-

$$\rho = 1.01\lambda\ f/\# \qquad (\ 115/D_{mm} \text{ arc secs })$$

and by examining the Airy disc diagram, commensurate with this separation, it is seen that this corresponds to a contrast difference of approximately 2 : 1.*



Contrast index $\kappa = \frac{r_2}{r_1}$

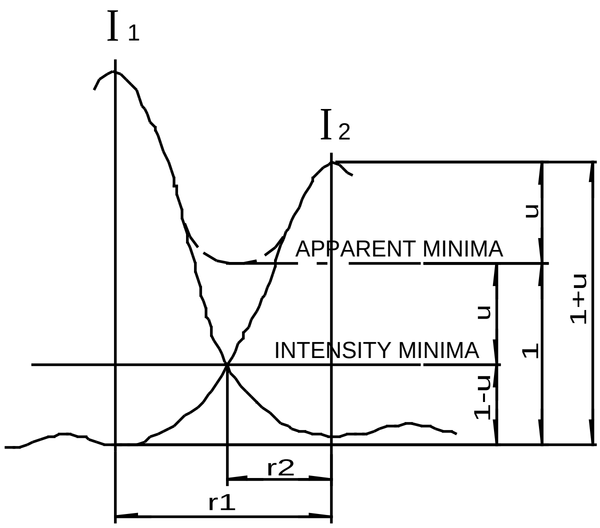
Contrast difference $\gamma = \frac{1}{1-\kappa}$

for Dawes' limit $\kappa = 0.55$ and $\gamma = 2.222$

** This corresponds to a 16.5% intensity minima between the maxima of the two patterns. This is because at normally encountered f/ratios (f/5 to f/20) no intensity gradient across the disc is perceptible.*

Note: Dawes empirical criterion states that a pair of mag. +2 stars, separation 4".56 are just resolvable with a 1-inch objective, and the figure stated by him to apply to two mag+6 stars seen in a 6-inch refractor.

The general case may be expressed as follows:-



Let $\kappa = \frac{r_2}{r_1}$

Intensity of Airy Disc Centres:-

$$I_1 \propto r_1^2$$

$$I_2 \propto r_2^2$$

Putting:-

$$\frac{I_1 - I_2}{I_1} = \psi^2$$

$$1 - \frac{I_2}{I_1} = \psi^2$$

$$\therefore \psi = \sqrt{1 - \left(\frac{r_2}{r_1}\right)^2} = \sqrt{1 - \kappa^2}$$

Contrast difference:-

$$\gamma = \frac{1}{1 - \sqrt{1 - \psi^2}}$$

$$\therefore \psi = \sqrt{1 - \left(1 - \frac{1}{\gamma}\right)^2}$$

$1 - \psi =$ minimum intensity difference

Resolution of
Airy discs
contrast ĸ
at image plane:-

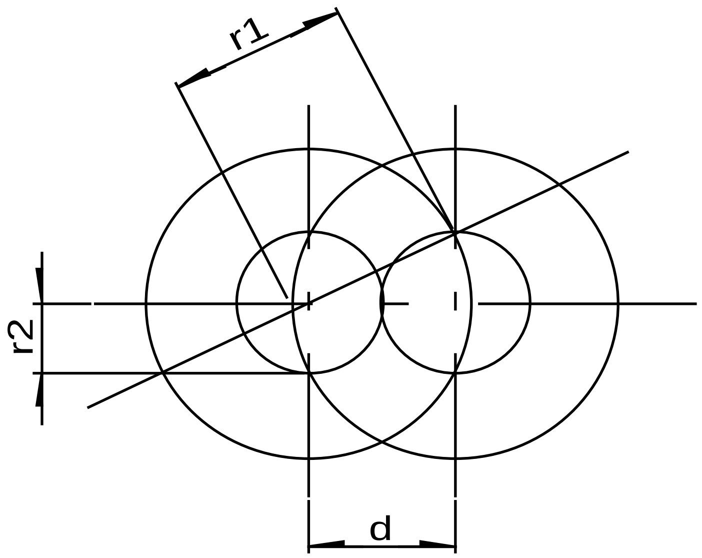
$$\rho = \delta.\lambda.\, f\,/\#$$

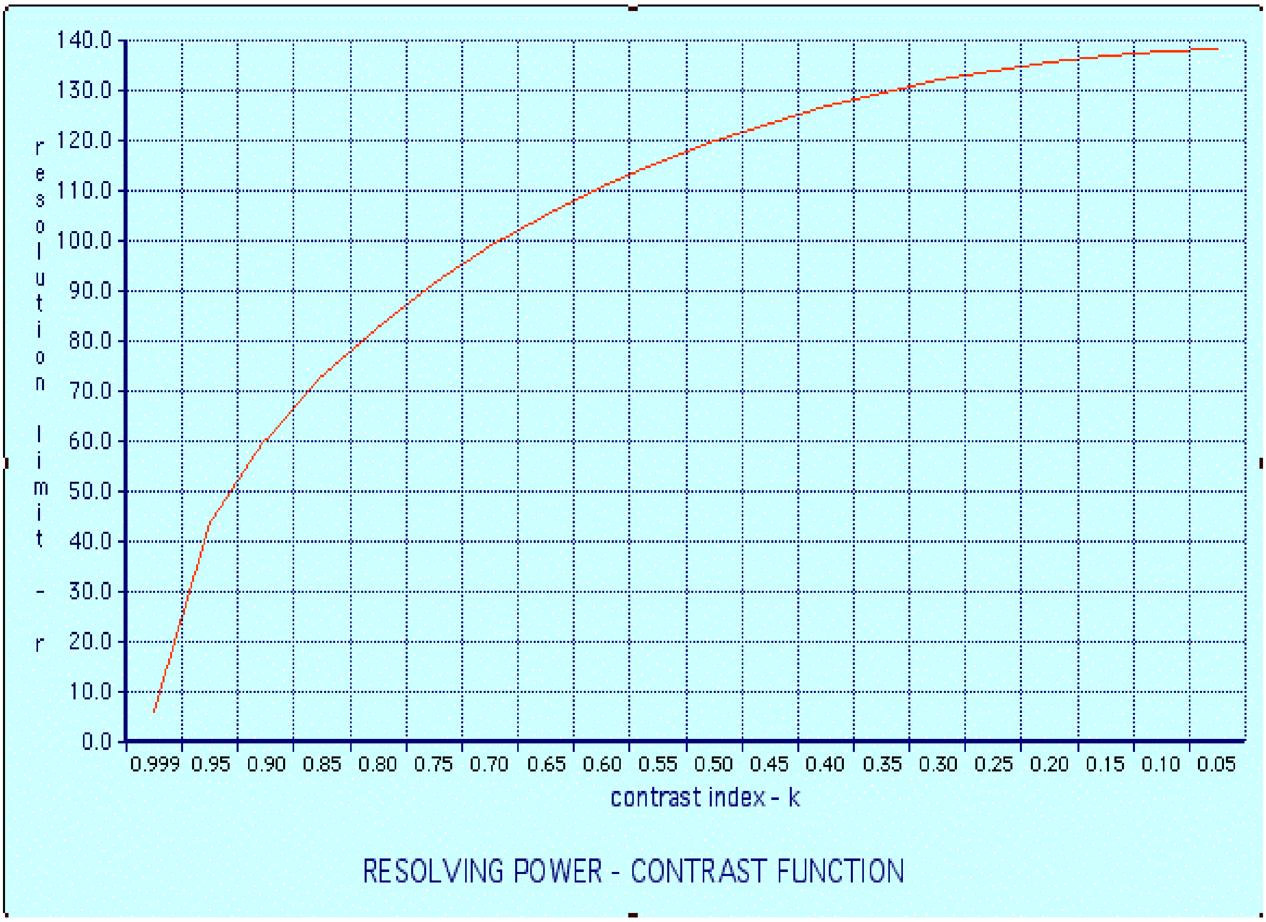
$$\delta \propto r_1.\psi$$

$$\therefore \rho \propto r_1.\psi.\lambda.\, f\,/\#$$

$$= c.r_1.\psi.\lambda.\, f\,/\#$$

$$\rho = 1.22\,\lambda\, f\,/\#\,\sqrt{\left(1-\kappa^2\right)}$$





The graph and table 1 show the relationship for varying contrast indices ranging from ĸ = 0.05 to 0.999 (5% to 1000:1)

Table 1. (see graph)

Resolution limit
for detail on
extended objects
contrast index Ⲕ
in the image plane:

k	r = n/D (mm)	rho	log gamma	notes
0.999	6.2	0.055	3.00000	y = 1000:1
0.99	20	0.172	2.00000	y = 100:1
0.98	28	0.243	1.69897	
0.97	34	0.297	1.52288	
0.96	39	0.342	1.39794	
0.95	44	0.38	1.29243	
0.90	61	0.53	0.99568	y = 10:1
0.85	73	0.64	0.82102	
0.80	83	0.73	0.69680	
0.75	92	0.81	0.60033	
0.70	99	0.87	0.52143	
0.65	105	0.93	0.45469	
0.60	111	0.98	0.39686	
0.55	116	1.02	0.34582	Dawes* limit
0.50	120	1.06	0.30016	Sparrow's limit
0.45	124	1.09	0.25885	
0.40	127	1.12	0.22113	
0.35	130	1.14	0.18642	
0.30	132	1.16	0.15428	
0.25	134	1.18	0.12436	
0.20	136	1.20	0.09637	
0.15	137	1.21	0.07007	
0.10	138	1.21	0.04528	
0.05	138	1.22	0.02182	Rayleigh limit

* *Sparrow's limit is a refinement of the Rayleigh limit in which the Airy disc centres are separated such that a contrast difference of 2 : 1 occurs. See Warren J. Smith, "Modern Optical Engineering" -- chapter dealing with "Resolution and contrast."*

From this it is evident that for extremely high contrast features resolution increases dramatically. For theoretical black lines upon a brilliant white ground, the resolution is almost a fifteenth Dawes' limit.[16]

It should be noted however that at best seeing on the earth's surface is no better than 0".15 to 0".2 arc and therefore all telescopes theoretically capable of resolution below this limit are "atmosphere limited" rather than diffraction limited.*

Resolution of unequal double stars

An interesting corollary to this modification of the Rayleigh formula is one governing the theoretical resolution of double stars differing markedly in brightness.[13] In this instance the separation of the Airy discs is proportional to their luminance, hence brightness and areas:-

$$\frac{Area\ \mathrm{Pr}imary}{Area\ Comes} = \frac{\frac{1}{4}\pi I_p^2}{\frac{1}{4}\pi I_c^2} = \left(\frac{I_p}{I_c}\right)^2 = \text{ brightness difference}$$

Therefore separation:- $r = \frac{I_p}{I_c}\rho'.\rho.\lambda.f/\#$

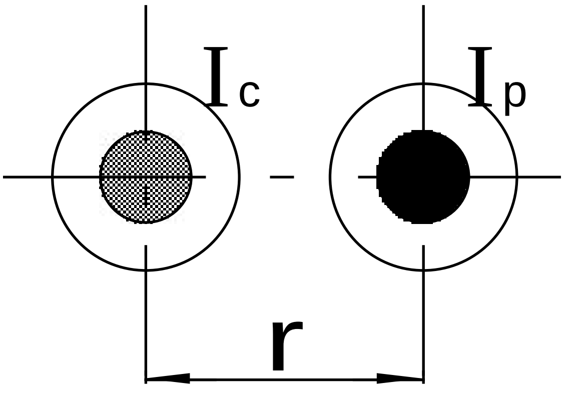
where $\rho'\ is\ \rho$ for the contrast difference factored to Dawes' limit

Brightness difference:-

$$\left(\frac{I_p}{I_c}\right)^2 = \left(\sqrt[5]{100}\right)^{0\cdot 1\Delta m}$$

where:- $\Delta m = m_c - m_p$

$$\therefore r = (100)^{0\cdot 1\Delta m}\, 1.01\,\rho'.\,\lambda.\ f/\#$$



** Generally seeing is nowhere near these limits. Typically one can anticipate seeing between 0".5 and 5" arc. Since I wrote this paper in 1979 adaptive optics have been developed that enable wavefront errors due to seeing to be, to some extent corrected.*

As a classic example consider αCMa (Sirius)
Primary -1.42; comes + 8.65, separation 9”.4 [1979]

$$\begin{aligned}\Delta m &= m_c - m_p \\ &= 8.65 - (-1.42) \\ &= 10.07\end{aligned}$$

Brightness difference = 10,000 approx

Taking a mean wavelength of 475nm for the primary, and a minimum contrast difference for complete resolution as 2 : 1 (Dawes’ limit) we have:-

Linear separation:

$$r = 100.1 \cdot 01.\rho'.0 \cdot 475.206265.10^{-3} / D_{mm}$$

$$r = 1396 / D_{mm}$$

(ρ' is computed from ρ for $\kappa' = 0.99$

($\gamma' = 100:1$) $\therefore \rho' = 0 \cdot 1410673$)

Therefore Sirius B should theoretically be completely resolved in a 150mm aperture.[8]

αCMi presents similar difficulties. The comites was first sighted by J.M. Schaeberle in 1896 with the great 36-inch Lick refractor at Mt. Hamilton; P.A. 320°, 4”.6 separation. I am unaware of anyone observing Procyon B with a lesser instrument although the following calculation indicates that it is theoretically possible to do so:-

$$\begin{aligned}\Delta m &= m_c - m_p \\ &= 10.80 - 0.35 \\ &= 10.45 \\ &= 100^{1.045}.1 \cdot 01.\rho'.\lambda.f/\# = \frac{1793.8}{D_{mm}} \\ &\cong \frac{1800}{D_{mm}} \quad (\kappa' - 0 \cdot 99187;\ \rho' = 0 \cdot 1272556)\end{aligned}$$

therefore at the time of discovery it should have been visible in only a 390mm instrument.

Both Otto Struve and S.W. Burnham searched for this object for many years without success using larger apertures.[9] A. Auwers published a computed orbital period of 40 years based upon proper motion irregularities in 1861.[10]

At the time of discovery the magnitude of the comites was estimated at + 13m_v giving a value $\Delta m = 12 \cdot 65$ from which $r = 2980/D_{mm}$ arc secs thereby requiring a 660mm aperture, and a 600mm aperture currently [sep 5” arc 1979].

It would be interesting to see if this were indeed possible. Double star observers using telescopes between 360mm and 600mm should search for this object at PA 300^o.

Care should be exercised when performing these double star calculations. Ensure the aperture deduced is able to show a star of the comites magnitude. As a rule of thumb the light gathering power of the instrument should be such that the faintest magnitude is 1.0m_v below that of the comes.

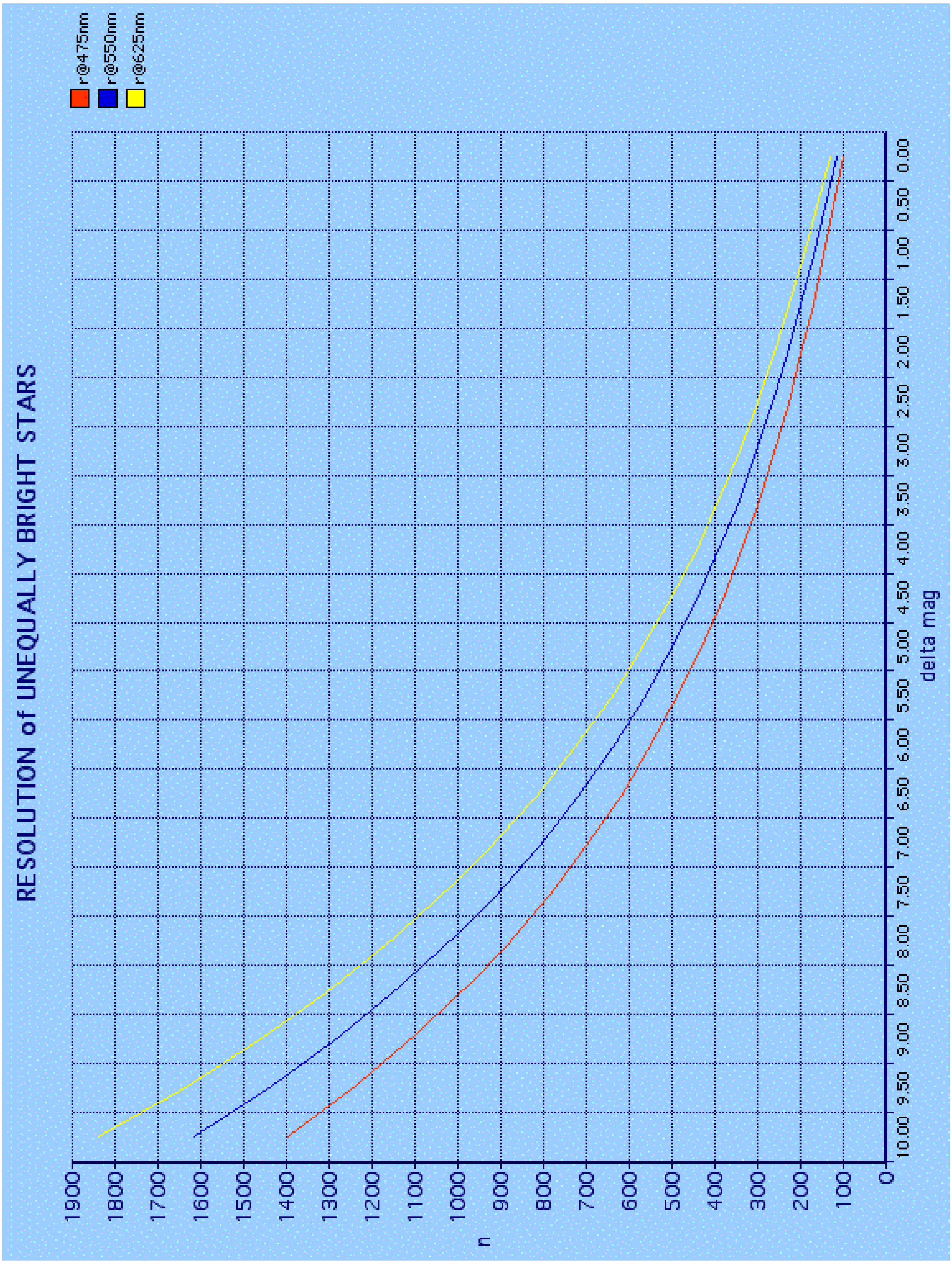
Table 2 and the accompanying graph show the relationship between resolution and aperture for double stars having magnitude differences Δm. Bear in mind that the wavelength chosen for doubles differing markedly in brilliance should be that of the primary. However when the pair are nearly equal in brightness but are distinctly different in colour a value for wavelength commensurate with their combined spectral characteristics would be more appropriate. It is also assumed the telescope is a good one and the seeing is perfect (Antoniadi I).

PTO

Table 2 (see graph overleaf)

Delta mv	r@475nm	r@550nm	r@625nm	(Ip/Ic)^2	100^0.1delt m	k'	rho'
10.00	1396	1616	1837	1.00000e+4	1.00000e+2	0.990	1.4106736e-1
9.50	1243	1440	1636	6.30957e+3	7.94328e+1	0.987	1.5817718e-1
9.00	1107	1282	1457	3.98107e+3	6.30957e+1	0.984	1.7733210e-1
8.50	986	1141	1297	2.51189e+3	5.01187e+1	0.980	1.9876403e-1
8.00	877	1016	1155	1.58489e+3	3.98107e+1	0.975	2.2272578e-1
7.50	781	904	1027	1.00000e+3	3.16228e+1	0.968	2.4949059e-1
7.00	694	804	914	6.30957e+2	2.51189e+1	0.960	2.7935021e-1
6.50	617	715	812	3.98107e+2	1.99526e+1	0.950	3.1261088e-1
6.00	548	635	721	2.51189e+2	1.58489e+1	0.937	3.4958604e-1
5.50	487	563	640	1.58489e+2	1.25893e+1	0.921	3.9058427e-1
5.00	431	499	568	1.00000e+2	1.00000e+1	0.900	4.3588989e-1
4.50	382	442	502	6.30957e+1	7.94328e+0	0.874	4.8573259e-1
4.00	337	391	444	3.98107e+1	6.30957e+0	0.842	5.4024048e-1
3.50	297	344	391	2.51189e+1	5.01187e+0	0.800	5.9936779e-1
3.00	261	302	344	1.58489e+1	3.98107e+0	0.749	6.6278319e-1
2.50	228	264	300	1.00000e+1	3.16228e+0	0.684	7.2969551e-1
2.00	198	230	261	6.30957e+0	2.51189e+0	0.602	7.9857687e-1
1.50	171	198	225	3.98107e+0	1.99526e+0	0.499	8.6670977e-1
1.00	146	169	192	2.51189e+0	1.58489e+0	0.369	9.2941246e-1
0.50	122	141	160	1.58489e+0	1.25893e+0	0.206	9.7862103e-1
0.00	99	115	130	1.00000e+0	1.00000e+0	0.000	1.0000000e+0

Resolution of unequally bright double stars:-



Aperture v atmosphere

Lowell was the first astronomer to repeatedly stress the importance of site instead of aperture. His comments on the subject are both amusing and instructive.[11]

"The sanction to any statement about "seeing" consists in some absolute test to which its relativity may be put. Hitherto the only test has been the *ipse dixit* of the observer graded if you will by what he sees but based on what he thinks he ought to see. One observer, for instance, will mark the seeing perfect while he is unable to detect detail which at another station is visible in air tabulated as mediocre and the world decides which of the two is right by abstract reference to the size of their object glasses, a standard only preferable to that of the diameters of their respective domes."

In the same discussion Lowell touched briefly on his conviction of the importance of locating observatories is arid regions:

"It is evident that to secure definition the air currents must be at a minimum and what there are of them as steady as possible. Now water in the air is the great unsettler. As dry a place as maybe is therefore the first *disideratum,* i.e. a desert. But as diurnal change in deserts is great we must have a shelter from the change or in other words an oasis in a desert. These conditions are fulfilled by the great pine oasis of Flagstaff in the midst of the Arizona desert. From the above we see incidentally why mountain tops will not do on the one hand nor anything situate above 40^0 of latitude on the other. For the roaring forties are quite disasterous to telescopy on land as they are to shipping at sea."[12]

That he showed a good deal of insight may be verified by the timely relevance of his remarks. It was not until almost a quarter of a century later that other professional astronomers began site testing in earnest.

This was the primary reason the pursuit of ever superior resolution in the drive for telescopes of greater and yet greater aperture at the dawn of the C20th fell foul of the expectations of Lord Rayleigh's findings.

Lowell, Pickering and Douglass spent over 10 years investigating sites as far afield as the Peruvian Andes to the Altai Scarp of the Moroccon Sahara. Lowell reasoned quite correctly that an elevated plateau in an arid, cool region, beyond the trade winds offered the best prospects for both clarity of skies and good seeing. His comments regarding mountain top sites may not be 100% correct, but there are very few exceptions.

What we have learned in the intervening period is that high altitude sites above a temperature inversion, alongside a cold ocean current, preferably on the western side of a continental landmass offer the most promising prospects. It has become evident however that nowhere on earth is seeing steadier than 0".15 to 0".2 arc and thus atmospheric turbulence forever remains an all pervading malaise to earth-bound observers employing large apertures. Even active optics cannot redress the deliterious effects and enable the full resolution of very large apertures to be realized.

A rule of thumb has grown up around this poorly evaluated concept, that the Rayleigh limit only adequately describes the expected resolving powers of diffraction limited telescopes less than about 300mm aperture, depending marginally on the site.

High contrast features, be they circular or linear $(\gamma \approx 25:1)$ may be resolved to the atmospheric limit with only a 200mm telescope, and even moderately low contrast features $(\gamma \approx 5:1)$ with a 415mm aperture. This is the reason apertures greater than this show very little extra detail upon extended objects. For example, assuming a hypothetical telescope located at a favourable site that occasionally enjoys 0".15 arc seeing, and an observer capable of detecting a minimum contrast difference of 5%, the minimum aperture required to show that detail is only 920mm. As far as visual lunar and planetary observation is concerned for a telescope located *anywhere* on the earth's surface, apertures greater than this are wasted. Their potential can never be realised.

As for lunar and planetary observation in the British Isles. The amateur observing here "enjoys" seeing no better than 0".25 and then only very infrequently. What is classed euphamistically as Antoniadi I in England and Scotland is usually 0".25 to 0".4 arc. The moist air reduces the lowest perceptible contrast difference to approximately 2 : 1, and therefore the maximum aperture one could usefully employ is 480mm at the best of sites (e.g. the South Downs and Salisbury Plain, the Wold and the Fylde Coast) to 300mm at fair coastal and hill sites. Table 3 and the accompanying graph on pages 17 & 18 of the summary show the relationship between wavefront distortion, contrast and aperture.

Summary

In conclusion object contrast at the image plane and wavefront distortion induced by atmospheric turbulence are the primary factors limiting resolution in a “perfect” telescope. To the visual lunar and planetary observer a small or medium aperture telescope of excellent quality will perform equally to an instrument possessed of a much larger aperture.

The resolution formula for the Rayleigh limit:-

$$\rho = 1\cdot 22\,\lambda\ f/\#$$

should be rewritten as:-

$$r = 1\cdot 22\sqrt{1-\kappa^2}\ \lambda\ f/\#$$ where κ is the contrast index

& $$\kappa = \frac{1}{1-\gamma}$$ where γ is the contrast difference

This may be employed to show the expected detail (contrast difference γ), visible in any particular telescope provided the telescope is excellent and the site a good one.

The formula may also be rewritten to show the expected aperture required to resolve double stars differing markedly in brightness:-

$$r = (100)^{0\cdot 1\Delta m}.1\cdot 01.\rho'.\lambda.\,f/\#$$

where $\rho' = \sqrt{1-\kappa'^2}$

& $$\kappa' = 1 - \frac{1}{\gamma'}$$

$$\Delta m = m_c - m_p$$

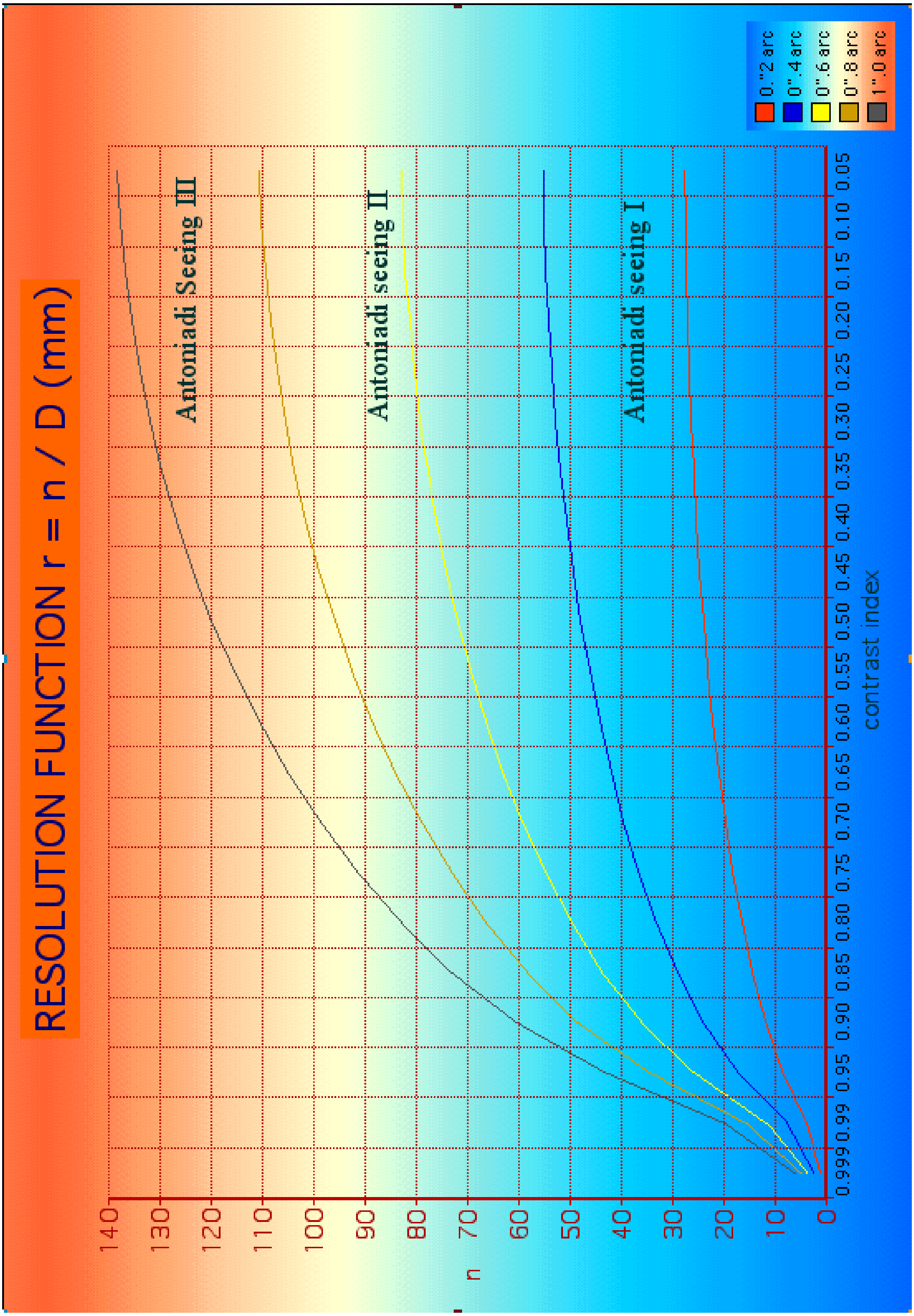
Both these formulae explain fully the various observational phenomenon witnessed by observers since the development of diffraction limited optics in the late C18th and Lord Rayleigh’s investigations of the mid C19th which sought to theoretically define precisely what limited a telescope’s resolution, but which in practice could only be applied to telescopes less than approximately 300mm aperture.

Appendix

Table 3 (see graph)

MINIMUM APERTURE to RESOLVE DETAIL CONTRAST INDEX κ at IMAGE PLANE for SEEING LIMIT

contrast index	resolution limit function: r = n/Dmm					comments
κ	0."2 arc	0".4 arc	0".6 arc	0".8 arc	1".0 arc	
0.999	1	2	4	5	6	(Sine target)
0.99	4	8	12	16	20	
0.95	9	17	26	35	43	(Cassini's division)
0.90	12	24	36	48	60	(Shadow transits)
0.85	15	29	44	58	73	(Lunar detail........
0.80	17	33	50	66	83	..........rhillen &
0.75	18	37	55	73	92	..........craterlets)
0.70	20	40	59	79	99	
0.65	21	42	63	84	105	(Sparrow's limit)
0.60	22	44	66	89	111	
0.55	23	46	69	92	116	(Dawes' limit)
0.50	24	48	72	96	120	
0.45	25	49	74	99	124	
0.40	25	51	76	101	127	(Planetary........
0.35	26	52	78	104	130	....
0.30	26	53	79	106	132	....
0.25	27	54	80	107	134	....
0.20	27	54	81	108	136	....
0.15	27	55	82	109	137	....
0.10	28	55	83	110	138	..............detail)
0.05	28	55	83	111	138	(Rayleigh limit)


RESOLUTION FUNCTION r = n / D (mm)
Antoniadi Seeing III
Antoniadi seeing II
Antoniadi seeing I
0."2 arc
0".4 arc
0".6 arc
0".8 arc
1".0 arc
n
contrast index
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
0.999
0.99
0.99
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05

References

1) Airy, George Biddell, A.M. "On the Diffraction of an Object-glass with Circular Aperture," Transactions of the Cambridge Philosophical Society, Vol. V. Pt. III, XII, 1835.

2) Dawes, William Rutter, Catalogue of Micrometrical Measurements of Double Stars (Introduction). (*Mem*. R.A.S., 35, No.4, 137).

3) Lowell, Percival, Mars, London 1895, v.,
Lowell, P., "Atmosphere: In its Effect on Astronomical Research", lecture text *ca*. spring 1897; L.O.A.

4) Pickering, W.H., "Climate as Related to Astronomical Observations", *Popular Astronomy*, 3 : 465 (1896), and Douglas, A.E., "Atmosphere, Telescope and Observer", Ibid, 5 : 64 (1897)

5) It is interesting to note that as late as 1971, an astronomer at the California Institute of Technology's Jet Propulsion Laboratory declared that "seeing" and its related phenomena, "have been so intensively studied in the last few years that they are now fairly well understood - but not by astronomers!"

See Andrew T. Young, "Seeing and Scintillation", Sky and Telescope, 42 : 139 (1971).

Young's article, incidentally, includes this passage which, except for its tentative tone, seems startingly Lowellian: "Evidentally, there is an optimum aperture to use if maximum resolution is wanted. With too small a telescope, the seeing is good (apart from image motion), but the telescope size limits the resolution. With too big a telescope, larger and stronger turbulent areas are included, and seeing limits the resolution. The aperture must be matched to the seeing conditions."

To which Lowell himself might have added: *Quad erat demonstrandum!*

References (cont.)

6) Phillips, T.E.R., & Steavenson, W.R., (eds), *Splendour of the Heavens* (Hutchinson, 1923).

7) Lowell, P., “Means, Methods and Mistakes in the Study of Planetary Evolution”. Text dated April 13, 1905, of a paper communicated to the Royal Astronomical Society, Nov. 15, 1905; L.O.A.

8) Burnham’s Celestial Handbook Vol. I., Burnham Jr., Robert, L.O.A., p 395; “In the winter of 1962, during a period of exceptionally good seeing, the visibility of the companion (α2CMa) was studied at Lowell Observatory with the 24-inch refractor (another superb Clark telescope) using an adjustable iris diaphragm over the objective. It was found that the faint star was most conspicuous with the aperture reduced to 18-inches, which helped to reduce some of the dazzling glare of the primary; it was still definite at 12-inches, difficult at 9-inches, and detectable at 6-inches only because its exact position was known. The tests were made with magnifications ranging from 200x to 900x. With the higher powers, it was possible to view the companion with Sirius itself placed entirely outside the field!”

9) Ibid., p450.

10) Ibid., p450.

11) op cit 3) Lowell, P., “Atmosphere”

12) Ibid.

13) Lewis, T. “On the Class of Double Stars which can be observed with Refractors of Various Apertures” (*Obs*., 37, No. 479, 378).
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14) Danjon, M.A., “L’Acuite Visuelle et ses Variations (*Rev. d’Opt*., 7, 205).

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16) Ibid., 2.6, p 49.