--- SHORT PAPER ---

# CALIBRATING the EFFECTIVE FOCAL LENGTH *of* CATADIOPTRIC CASSEGRAINS with MOVING PRIMARY FOCUSSING

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**Catadioptric Cassegrain telescopes with moving primary focussing have a variable focal length. Unlike conventional fixed focal length telescopes, focus is effected by adjusting the separation of the primary and secondary mirrors. This is accomplished by moving the primary along a spring loaded axial sleeve mounted through the central perforation. This in turn alters the effective focal length, and focus occurs when the Cassegrain focal plane is brought into coincidence with the accessory located on the backplate.**

The method possess the advantage that any accessory may be adjusted to focus, regardless of its optical path distance, and a large difference in back focus may be accommodated by only a small movement of the primary.

What is not generally appreciated is that the effective focal length increases in far greater proportion than the increase in back focus, and the effective focal length quoted by the manufacturer only applies at the backplate.

Calibrating the variable effective focal length is awkward, but a useful property can be utilized that simplifies matters.

Referring to fig_1 which schematically depicts any catadioptric Cassegrain, it can be seen that:

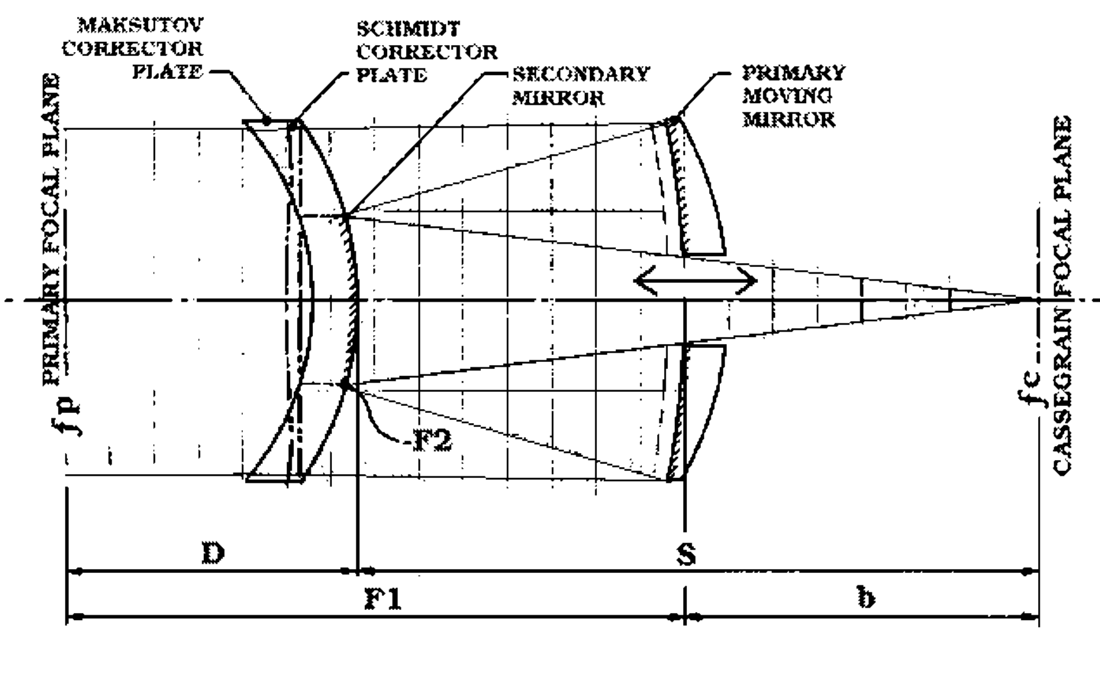


*fig.1*

$$F_1 + b = S + D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$\text{but} \quad A = S.D^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

$$\text{where} \quad A = \text{amplification by secondary}$$

$$\& \quad S = F_2(A-1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

$$\text{where} - F_2 = \text{secondary focal length}$$

$$\text{from which} \quad b = F_2(A-1) + F_2(A-1).A^{-1} - F_1 \ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

$$\text{or} \quad b = (D - F_2) - F_2^2(D - F_2)^{-1} - F_1 \ldots\ldots\ldots\ldots\ldots\ldots (v)$$

$$\& \quad A = \left((F_1 + b) + \left((F_1 + b)^2 + 4F_2^2\right)\right)^{0.5}.0.5.F_2^{-1} \ldots\ldots\ldots (vi)$$

$$\& \quad \frac{dA}{db} = A^2(A^2+1).F_2^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (vii)$$

$$\frac{d^2A}{db^2} = -A^3.0.5.F_2^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (viii)$$

$$\frac{db}{dD} = A^2 + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ix)$$

$$\frac{d^2b}{dD^2} = -2F_2.A^{-3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (x)$$

***(for derivation of these equations refer to the appendix)***

In practice, where **A** is in the working range **5<A<10**, the rate of change of **A** with backfocus **(b)** is almost constant. The inverse slope of $\frac{d^2A}{db^2}$ is almost zero, permitting **A** to be represented by a linear expression:

$$A = A_0 + \frac{dA}{db}.b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xi)$$

$$EFL = F_1\left(A_0 + \frac{dA}{db}.b\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xii)$$

$$E f/\# = f_p\left(A_0 + \frac{dA}{db}.b\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xiii)$$

$$\text{where} \quad EFL = \text{effective focal length}$$

$$E f/\# = \text{effective focal ratio}$$

$$f_p = \text{primary focal ratio}$$

**$A_0$** may be determined from equation **(vi)** when **b = 0**

The difficulty one encounters in calibrating **EFL** against backfocus is determining initial values of **S**; **$F_1$**; **A & b**. I employed an optical comparator and a long focus ocular calibrated on a focometer to ascertain the **EFL** at a known distance behind the backplate. The **EFL** at the backplate can then be determined using equation **(viii)**. Values for **($F_1$ - D)** and **S** at the backplate may be measured directly. The primary focal length may be taken from the manufacturer's data sheets.

Example: Quantum 6 Maksutov-Cassegrain manufactured by Optical Techniques, Philadelphia from manufacturer's data sheets:

**nominal focal ratio at backplate : Ef/no. = f / 15**
**nominal EFL at backplate = 90" (2286mm)**
**focal ratio of primary : $f_p$ = f / 2.5**
**nominal amplification at backplate : $A_{nom}$ = x 6**
**primary focal length : $F_1$ = 15" (381mm)**

measurements:

**separation of secondary and backplate : S = 19 1/2" (495.3mm)**
**separation of primary and secondary : $F_1$ - D = 11 3/4" (298.45mm)**

$$\text{from } (i) \quad b = S - (F_1 - D) = 7\tfrac{3}{4}" \quad (196.85mm)$$

$$\text{from } (ii) \quad D = S.A^{-1} = 3\tfrac{1}{4}" \quad (82.55mm)$$

$$\text{from } (iii) \quad -F_2 = -S(A-1)^{-1} = -3.9" \quad (-99mm)$$

$$\text{from } (iv) \quad \text{when} \quad b = 0; \quad A_0 = \left(F_1 + \left(F_1^2 + 4F_2^2\right)\right)^{0.5} .0.5.F_2^{-1} = 4$$

$$\text{from } (vii) \quad \frac{dA}{db} = 0.25$$

$$\text{from } (viii) \left(\frac{d^2A}{db^2}\right)^{-1} = 0.036$$

$$\text{from } (ix) \quad \frac{db}{dD} = 37$$

$$\text{from } (x) \quad \frac{d^2b}{dD^2} = 0.025$$

$$\text{defining} \quad (xi) \quad A = 4 + 0.25$$

$$\text{defining} \quad (xii) \quad EFL = 60 + 3.75$$

$$\text{defining} \quad (xiii) \quad Ef/\# = 10 + 0.625b$$

True focal length at backplate:

$$EFL = D.F_e.e_p^{-1}$$

$$where \quad D = objective\ aperture$$

$$F_e = ocular\ focal\ length$$

$$e_p = exit\ pupil$$

using ocular, calibrated focal length 3".1 (78.75mm), the exit pupil was measured using an optical comparator, and found to be 5.4mm.

Therefore:

$$EFL = 6.78.75.5.4^{-1} = 87''.5 \quad (2222.5mm)$$

Numerical solutions to (xi); (xii) & (xiii) were plotted, the abscissae adjusted to the calibration at the backplate (ref. fig_2).

The calculated and calibrated values are listed in the table below:

| D | deltaD | A | deltaA | b | delta b | EFL | delta EFL | corrected EFL | calibrated EFL | Sigma |
|---|---|---|---|---|---|---|---|---|---|---|
| 3.25 | 0.05 | 6.0 | 0.5 | 7.75 | 2.00 | 90.00 | 7.50 | 87.50 | 87.50 | 0.00 |
| 3.30 | 0.05 | 6.5 | 0.6 | 9.75 | 2.35 | 97.50 | 9.00 | 95.00 | 95.00 | 0.00 |
| 3.35 | 0.05 | 7.1 | 0.7 | 12.10 | 2.82 | 106.50 | 10.50 | 103.60 | 103.50 | 0.10 |
| 3.40 | 0.05 | 7.8 | 0.9 | 14.92 | 3.43 | 117.00 | 13.00 | 114.50 | 114.30 | 0.20 |
| 3.45 | 0.05 | 8.7 | 1.1 | 18.35 | 4.28 | 130.00 | 16.25 | 127.50 | 126.70 | 0.80 |
| 3.50 | 0.05 | 9.8 | | 22.63 | | 146.25 | | 143.75 | 142.50 | 1.25 |

Note both the change of backfocus **(b)** and effective focal length **(EFL)** with change of **D**, and the small error in the corrected values of **EFL** determined from **(xii)**, increasing progressively for increasing values of **b**.

Note how, for only a 1/4 inch (6.35mm) movement of the primary mirror, the backfocus increases by 14 7/8 ins. (378mm), and the effective focal length increases by 55 ins. (1397mm).

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# Appendix

Derivation of terms:

$$F_1 + b = S + D \text{.......................................................} (1)$$

$$thus: \quad b = S + D - F_1$$

$$S = F_2(A-1) \text{.......................................................} (2)$$

$$D = S.A^{-1} \text{..............................................................} (3)$$

$$\therefore \quad b = F_2(A-1) + F_2\frac{(A-1)}{A} - F_1 \text{..........................} (4)$$

$$Q \quad D = F_2\frac{(A-1)}{A}$$

$$\left[F_2 = \frac{S.D}{(S-D)}\right]$$

$$A = \frac{-F_2}{A}(D - F_2)$$

$$b = (D - F_2) - F_2^2(D - F_2) - F_1 \text{...............................} (5)$$

$$rearranging\ (4): \quad F_2.A^2 - (F_1 + b).A - F_2 = 0$$

$$\therefore \quad A = \frac{1}{2F_2}\left[(F_1 + b) + \sqrt{(F_1 + b)^2 + 4F_2^2}\right] \text{............} (6)$$

$$rewriting\ (4): \quad b = F_2.A - \frac{F_2}{A} - F_1$$

$$\therefore \quad \frac{db}{dA} = F_2 + \frac{F_2}{A^2}$$

$$\therefore \quad \frac{dA}{db} = \frac{\left[A^2(A^2+1)\right]}{F_2} \text{.............................................} (7)$$

$$\& \quad \frac{d^2b}{dA^2} = \frac{-2F_2}{A^3}$$

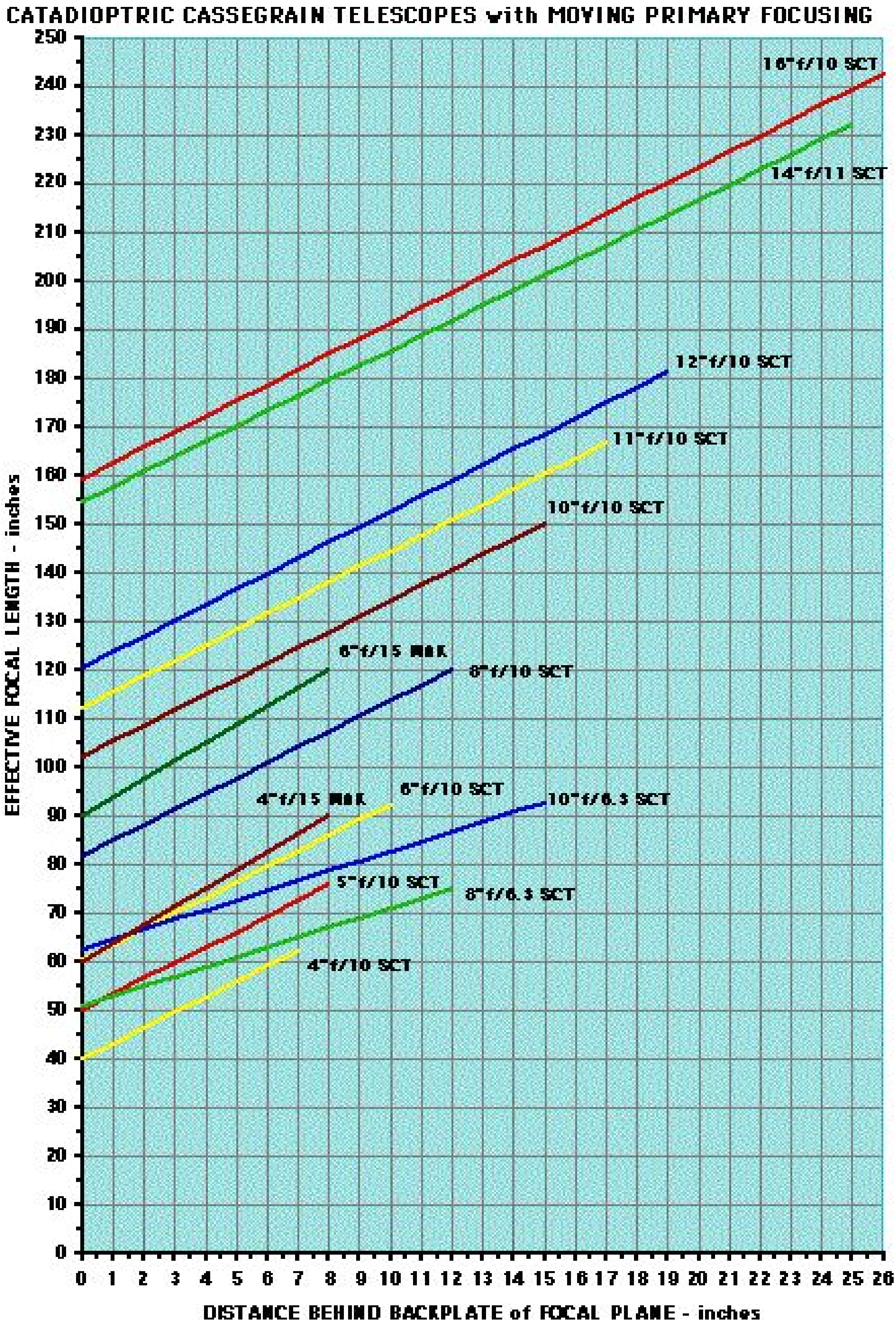
$$\therefore \quad \frac{d^2A}{db^2} = -\frac{A^3}{2F_2} \text{....................................................} (8)$$

$$rewriting\ (5): \quad b = (D - F_2) - \frac{F_2^2}{(D - F_2)} - F_1$$

$$let \quad k = D - F_2$$

$$\therefore \quad \frac{db}{dD} = \frac{db}{dk}.\frac{dk}{dD} = 1 + \frac{F_2^2}{(D - F_2)} = 1 + A^2 \text{................} (9)$$

$$\& \quad \frac{d^2b}{dD^2} = \frac{d^2b}{dk^2}.\frac{d^2k}{dD^2} = \frac{-2F_2^2}{(D - F_2)^3} = \frac{-2F_2}{A^3} \text{...............} (10)$$


CATADIOPTRIC CASSEGRAIN TELESCOPES with MOVING PRIMARY FOCUSING
EFFECTIVE FOCAL LENGTH - inches
DISTANCE BEHIND BACKPLATE of FOCAL PLANE - inches
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
16"f/10 SCT
14"f/11 SCT
12"f/10 SCT
11"f/10 SCT
10"f/10 SCT
6"f/15 MAK
8"f/10 SCT
4"f/15 MAK
6"f/10 SCT
10"f/6.3 SCT
5"f/10 SCT
8"f/6.3 SCT
4"f/10 SCT

*f* **ig.2**